Oscillations := The to-and-fro motion of a body around a fixed point is called oscillation.

For example := the motion of a simple pendulum, a steel strip fixed in a clamp and a mass suspended from a spring.

Periodic Motion := The motion of a body which repeats itself in equal interval of time is called Periodic motion.

For example

(i) The second's hand which of a watch repeats its motion on the dial in every 1 minute.

(ii) In 24 hours the earth completes its periodic rotation.

(iii) The simplest periodic vibratory motion is called simple-harmonic motion.

Lecturer : [redacted]
M.Sc Maths/Electronics

Simple Harmonic Motion :-

The motion of a body in which acceleration is directly proportional to the displacement but opposite in direction to the displacement and is always directed towards the mean position is called simple-harmonic motion.

Mathematically :-

If x is the displacement of the body from the mean position and F is the restoring force then

$$F \propto -x$$

$$\Rightarrow F = -Kx \quad \text{------} \quad (1)$$

Where K is a constant of proportionality.

The negative sign (–) shows the opposite direction.

But from Newton's IInd law, we have

$$F = ma \quad \text{------} \quad (2)$$

Compare Eq (1) and Eq (2), we get

$$ma = -Kx$$

$$\Rightarrow a = -\frac{K}{m}x \quad \text{———} \quad (3)$$

Since K and m both are constants, so

$$\therefore a = -\text{constant} \cdot x$$

$$\Rightarrow a \propto -x \quad \text{———} \quad (4)$$

This is the equation of Simple-harmonic motion.

EXAMPLE :- (i) The motion of a simple pendulum

(ii) The motion of a mass attached to elastic spring.

Characteristic of Simple Harmonic Motion

(i) A body oscillates about a central or mean position.

(ii) Acceleration produced by restoring force is always directed towards mean position.

(iii) Simple harmonic motion is always in a straight line.

(iv) The acceleration is directly proportional to the displacement but opposite in direction.

(v) The potential energy is maximum at extreme position and Kinetic energy is minimum at extreme position. The potential energy is minimum at mean position and Kinetic energy is maximum at mean position. But total energy of the system remains constant.

(vi) The acceleration is maximum at extreme position and minimum at mean position.

(vii) The velocity is minimium at extreme position maximum at mean position.

Lecturer: [redacted]

M.Sc Maths / Electronics

[redacted]

(i) Vibration := One complete round trip of the body is called vibration.

(ii) Amplitude := The maximum displacement of the body on either side of mean position is called amplitude.

(iii) Displacement := The distance of a body at any instant from the mean position is called displacement.

(iv) Time-Period := The time required to complete one vibration or oscillation is called time-period. It is denoted by T.

$$T = \frac{1}{f}$$

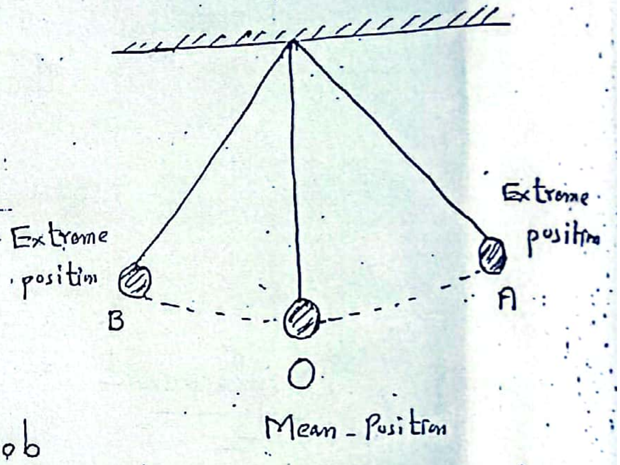
(v) Frequency := The number of vibrations completed by a body in one second is called frequency. It is denoted by f. Its unit is hertz (Hz).

$$\therefore f = \frac{1}{T}$$

Simple Pendulum :=

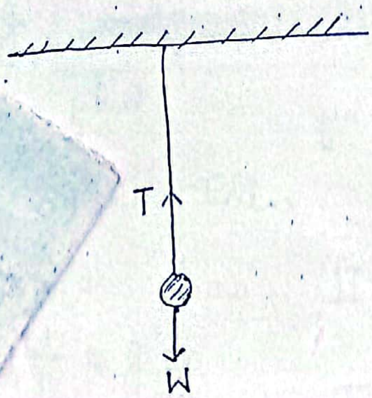
Simple Pendulum consists of a small metallic bob which is suspended from a support with the help of inextensible string.

The motion of a simple pendulum is an example of simple-harmonic motion.

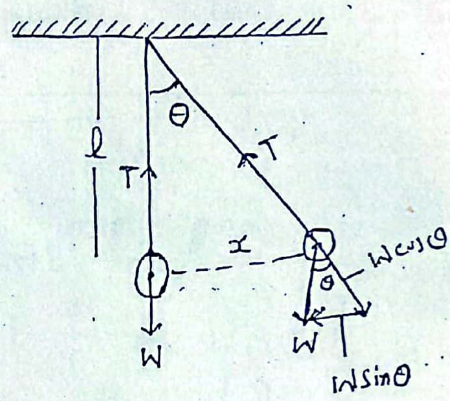


If we displace the bob from the mean position "O" and is brought to the extreme position "A" and then release, it will start vibrations.

Two forces are always acting on the bob. One is its weight and other is tension in the string.



The weight force can be resolved into two rectangular components. i.e $W\cos\theta$ and $W\sin\theta$.



One component ($W\cos\theta$) is along the string in a direction opposite to the tension in a string. The second component ($W\sin\theta$) is perpendicular to the string and is always directed towards O. The first component ($W\cos\theta$) balances the tension in the string and there is no motion along the string. The second component ($W\sin\theta$) is horizental and directed towards the mean position. It is this component which produces oscillation.

$$\therefore F = -W\sin\theta$$

-ve sign shows the opposite direction

$$ma = -mg\sin\theta$$

$$a = -g\sin\theta \quad \text{——— (1)}$$

From the figure, we have

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin\theta = \frac{x}{l}$$

Put this value in Eq ①, we get

$$① \Rightarrow a = -g \cdot \frac{x}{l}$$

$$\Rightarrow a = -\frac{g}{l}\, x \quad \text{———— ②}$$

Since g and l are constants, so

$$\therefore a = -(\text{constant}) \cdot x$$

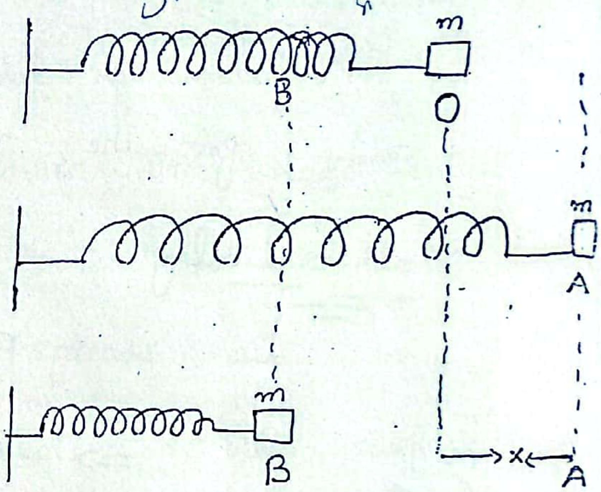
$$\Rightarrow a \propto -x \quad \text{———— ③}$$

This equation shows that motion of a simple-pendulum is an example of simple-harmonic motion.

Oscillations of a mass attached to a Spring

Consider a spring whose one end is attached to a fixed support and other end is attached to a mass m.

$\sin\theta = \frac{\text{mp}}{\text{Hypt}}$



When a spring is slightly stretched or compressed and is then released, it will start oscillations. Since motion of a mass attached to a spring is simple harmonic motion.

When mass m is released from extreme position "A". The mass m moves towards mean position "O". with decreasing acceleration and increasing speed (velocity). At O the acceleration is zero and speed is maximum.

The moving mass does not stop at O. Because of inertia, the mass m continues to move. The velocity is gradually decreased until the mass stops at the extreme position B. Under the action of restoring force, the mass m moves towards "O". On reaching O, its speed is maximum and because of inertia, the mass m over-shoots its mean position O. Eventually it reaches "A" and is then ready for the next vibration.

Mathematically :-

$$F \propto -x$$

$$\Rightarrow F = -Kx \quad \text{--- (i)}$$

Where K is a constant of proportionality.

-ve sign shows the opposite direction.

But from Newton's IInd law of motion

$$F = ma \quad \text{--- (2)}$$

Compare Eq (1) and Eq (2), we get

$$ma = -Kx$$

$$\Rightarrow a = -\frac{K}{m}x$$

$$\Rightarrow a = -(\text{constant})\, x$$

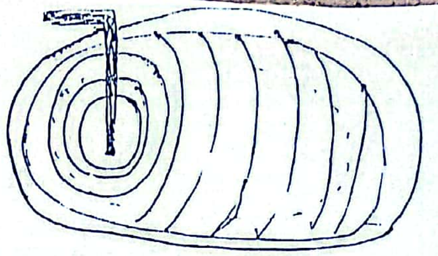
$$\Rightarrow a \propto -x$$

This equation shows that motion of a mass attached to a spring is SHM.

Waves := Wave is a disturbance that propagates through a medium and can carry energy but cannot carry mass.

Explanation := When a stone is dropped into a pool of water. A disturbance is created where the stone enters the water. The disturbance is not confined to that place alone. It spreads out in the form of circular wavelets on the surface of water. The water does not itself move outward from the point of disturbance. It merely rises and fall locally.

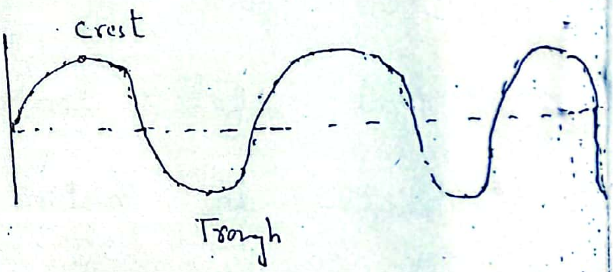
Observe the motion of pieces of paper floating on surface. When a wave reaches a floating objects, it begin to vibrate up and down. Their vibratory motion stops when the wave passes by the floating objects. The floating objects do not move with the waves. They simply vibrate locally. The vibration of floating objects is due to the vibratory motion of the particles of water.



Waves on surface of water.

Waves can also be produced and propagate in solid. Take a long string and mark it with the colours at equal spacing. Attach one end of the string to a hook in a wall. Hold the other end in your hand and stretch the string tight in the horizental position. Now give a sudden vertical jerk to the end, and move it up and down continuously and regularly.

You will see that a number of continuous waves are produced.



Such a set of waves is called a wave train.

If the coloured marking are carefully observed, as the waves travel down the string. You will see that they are vibrating about their mean position. Some part of the string are above mean position is called crest and some part of the string are below the mean position is called trough.

Waves as Carriers of Energy

The energy can be transferred from one place to another by the following three methods.

(i) Motion of a Macroscopic body

In this method energy is transferred from one place to another by complete motion of macroscopic body from one place to another.

Example :- (i) The energy carried by a moving truck

(ii) A projected stone

(iii) blowing winds etc.

(ii) Particle transmission :-

In this method, energy is transferred from one place to another due to collision of molecules of the medium.

EXAMPLE :- (i) Transfer of electrical energy

(ii) conduction of heat through a metal bar etc.

(iii) Wave Motion :=

In this method energy is transferred from one place to another by means of a wave. In wave motion, energy is transferred through the medium, although the particle of the medium are not transmitted.

EXAMPLE := (i) The sound of the school bell.

(ii) The light and heat from the sun reaches us by means of wave.

(iii) Water waves.

Mechanical Waves := The waves which require a medium for their propagation is called mechanical waves

e.g sound waves.

Electro-magnetic Waves := The waves which do not require medium for their propagation is called electro-magnetic waves.

e.g := light waves.

Kinds of Waves :=

There are two types of waves.

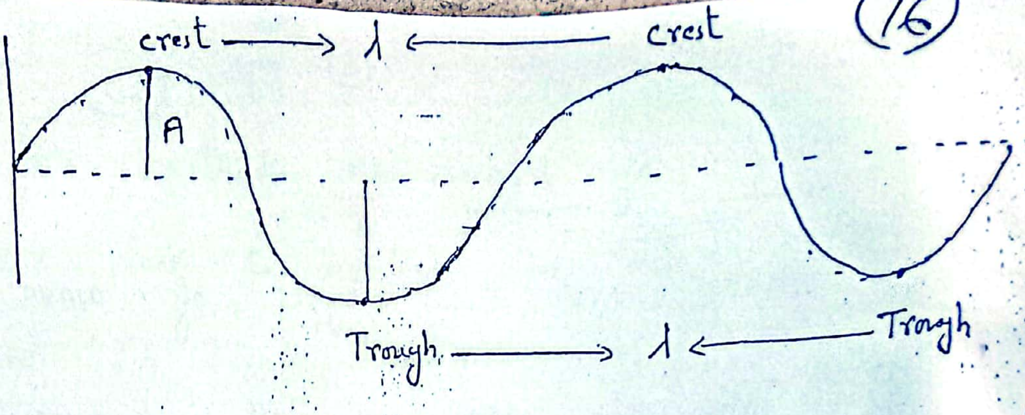
(1) **Transverse Waves** := The waves in which particle of the medium vibrate about their mean position perpendicular to the direction of propagation of waves are called transverse waves.

The waves on surface of water are transverse waves. They can be produced in a strings, rods and other solid media.

For example: if the free end of horizental cord is moved up and down in a simple harmonic manner. then transverse waves passing through a medium form crest and trough. The distance between two consecutive crests or troughs is called wave-length. It is denoted by λ (lambda).

A crest and trough joins to make one complete wave.

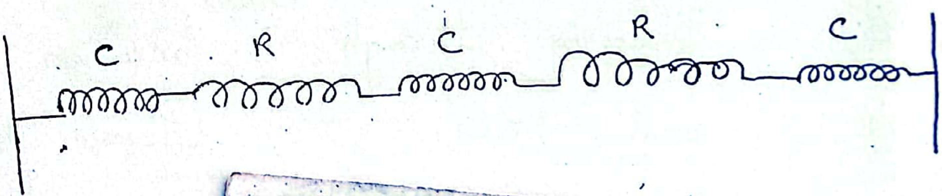
The amplitude of a particle of the medium of propagation is called amplitude of the wave.



Transverse - Waves

(2) Longitudinal Waves :- The waves in which particle of the medium vibrate about their mean position parallel to the direction of propagation of waves are called longitudinal waves.

The sound waves and waves in a spring are longitudinal waves.



Velocity of Waves :- The distance travelled by a wave in unit time is called velocity of wave.

Mathematically :-

$$V = \frac{S}{t} \quad \text{——— (1)}$$

Put $S = \lambda$ and $t = T$ in Eq (1), we get

$$(1) \Rightarrow \quad V = \frac{\lambda}{T}$$

$$\Rightarrow \quad V = \lambda \cdot \frac{1}{T}$$

$$\Rightarrow \quad V = \lambda f \qquad \because f = \frac{1}{T}$$

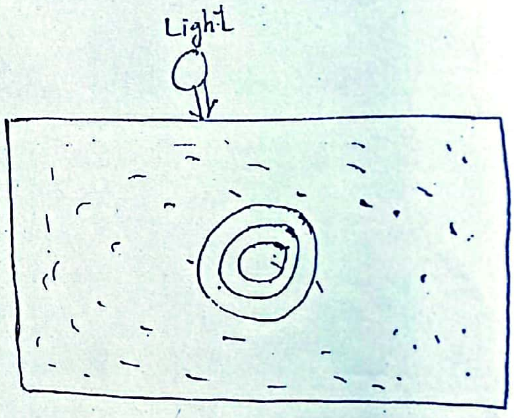
$$\Rightarrow \quad V = f\lambda$$

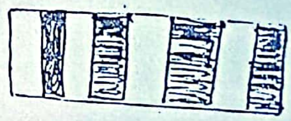
Ripple Tank And Demonstration of Wave Properties

A ripple tank consists of a rectangular tray containing water, fitted with a glass bottom. A light bulb is fixed above the tray and a viewing screen is placed well below the tray.

Various types of waves can be produced in the tank by disturbing the water in different waves. A circular pulse can be produced by simply dipping fingers into the water. Straight periodic waves can be generated by dipping a straight rod periodically into the water.

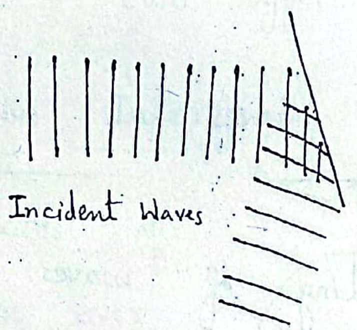
When waves are produced, the crests converge the light into the screen to form a bright bands while troughs diverge the light into the screen to form dark bands.





Reflection of Waves :=

Waves are reflected when an obstacle is placed in their path. In a ripple tank reflection can be demonstrated by placing an upright surface in water.



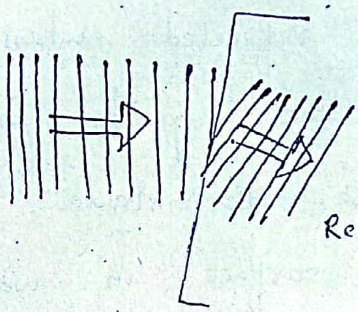
Refraction of Waves :=

def
(When waves enter from a rarer medium into a denser medium, they bend away slightly from their straight path is called Refraction of waves.)

The speed of wave is relatively greater in a rarer medium than in a denser medium.

In a ripple tank, place a plastic sheet in the bottom portion of the tray. The incident waves refract at the edge of the plastic sheet.

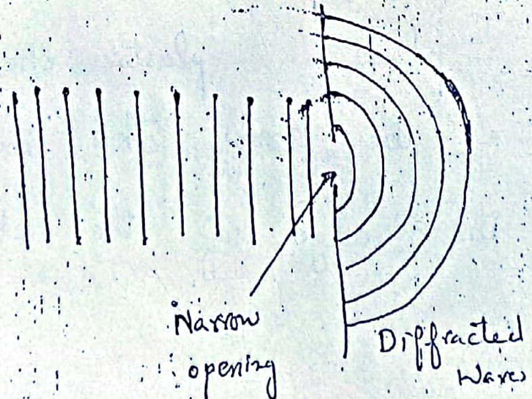
So the speed of waves in the deeper water is greater than their speed in the shallower water.



Diffraction of Waves :=

The spreading of waves occurs when passed through a small opening is called diffraction of waves.

Bring two obstacles close together in a ripple tank and leave a small opening between them. Generate waves in the ripple tank. You will observe that they diffract around the corners of the opening.

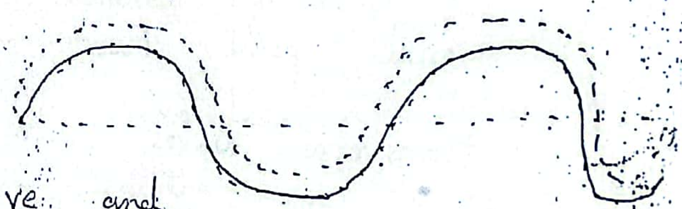


Interference of Waves

The superposition or overlapping of two or more than two waves with each other is known as interference.

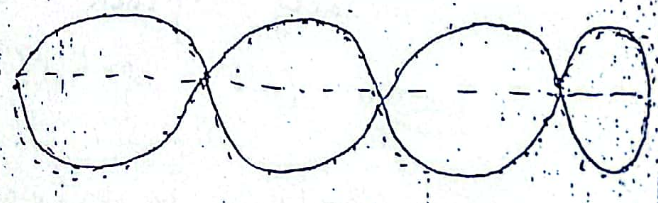
There are two types of interference

(1) Constructive Interference :=



When the crest of one wave come over the crest of other wave and trough of one wave come over the trough of other wave, then amplitude of the resultant wave will increase. This type of interference is called constructive interference.

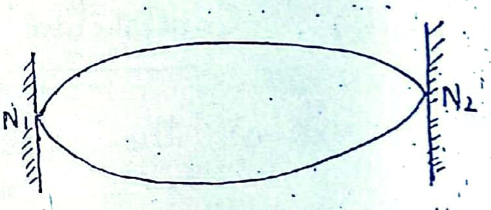
(2) Destructive Interference :=



When crest of one wave come over the trough of other wave and trough of one wave come over the crest of other wave, then amplitude of resultant wave is zero. This type of interference is called destructive interference.

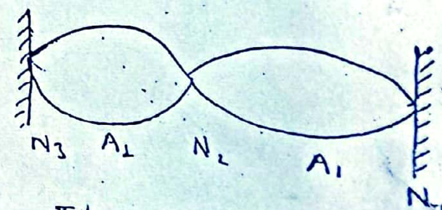
<u>Standing Waves</u> :- When two waves having same frequency and amplitude but opposite in direction and overlap (superimpose) each other, then standing or stationary waves are produced.

<u>Explanation</u> :- Take a steel string of length about 1 meter. Stretch it tightly and clamp it at its two ends. Pluck the string at its mid point. Two set of transverse waves are produced,

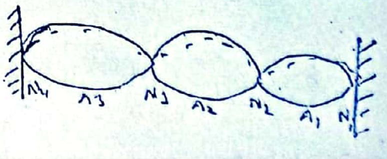


one will move to the left end of the string and the other towards the right end. When these waves reach the clamped ends, they are reflected back. The incident and the reflected wave trains causes the string to vibrate as a whole in the form of single loop.

Similarly pluck the string at one-quarter ($\frac{1}{4}$) of its length. The string now vibrates in two loops.



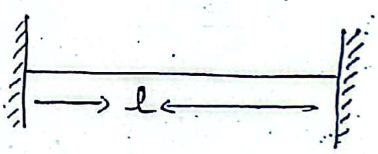
Also pluck the string at one-sixth ($\frac{1}{6}$) of its length. It will oscillate in three loops.



NODE := The portion of a standing wave in which the particle of medium are at rest, called node, denoted by N.

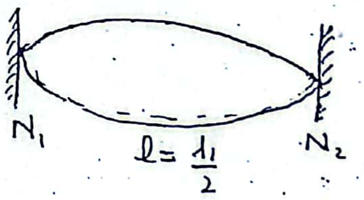
Anti-Node := The portion of a standing wave in which the particle of medium vibrate with maximum amplitude, called anti-node. It is denoted by A.

Mathematically := Let us consider a string of length l. If the string is plucked at the centre.



Since

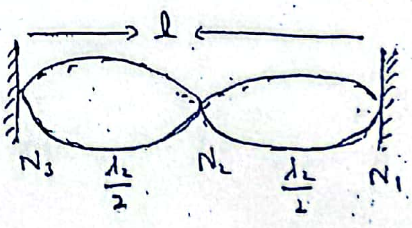
$$l = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2l \quad \text{--- (1)}$$



$$\because \quad V = f_1 \lambda_1 = f_1 (2l)$$

$$\Rightarrow f_1 = \frac{V}{2l} \quad \text{--- (A)}$$

If the same string is plucked at $\frac{1}{4}$ of its length, then



$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{\cancel{2}\lambda_2}{\cancel{2}}$$

$$l = \lambda_2$$

$\because \quad v = f_2 \lambda_2$

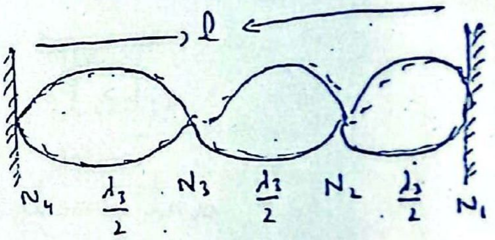
$\Rightarrow \quad v = f_2 l$

$\Rightarrow \quad f_2 = \frac{v}{l} = \frac{2v}{2l}$

$\Rightarrow \quad f_2 = 2\left(\frac{v}{2l}\right)$

$\Rightarrow \quad f_2 = 2f_1$ —— (B) $\quad \because \quad f_1 = \frac{v}{2l}$

Similarly.



$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$

$l = \frac{3\lambda_3}{2} \Rightarrow 3\lambda_3 = 2l$

$\Rightarrow \quad \lambda_3 = \frac{2l}{3}$

$\because \quad v = f_3 \lambda_3 = f_3\left(\frac{2l}{3}\right)$

$\Rightarrow \quad f_3 = \frac{3v}{2l} = 3\left(\frac{v}{2l}\right)$

$\Rightarrow \quad f_3 = 3f_1$ —— (C) $\quad \because \quad f_1 = \frac{v}{2l}$

From eq (A), eq (B) and eq (C), we have

$f_n = n f_1$

Numerical Problem

1

no. of vibrations $= n = 10$

time $= t = 1$ sec

Time-Period $= T = ?$

Frequency $= f = ?$

Since we know that

$$f = \frac{n}{t} = \frac{10}{1}$$

$$\Rightarrow \boxed{f = 10\ Hz}$$

Similarly

$$T = \frac{1}{f} = \frac{1}{10}$$

$$\Rightarrow \boxed{T = 0.1\ sec}$$

2

Frequency $= f = 2.5\ KHz = 2.5 \times 10^{3}\ Hz$

Time-Period $= T = ?$

Since we know that

$$T = \frac{1}{f} = \frac{1}{2.5 \times 10^{3}}$$

$$\Rightarrow T = 0.4 \times 10^{-3} = 4.0 \times 10^{-1} \times 10^{-3}$$

$$\Rightarrow \boxed{T = 4.0 \times 10^{-4}\ sec}$$

(3)

Time-Period $= T = 1.6 \times 10^{-15}$ sec

Frequency $= f = ?$

Since we know that

$$f = \frac{1}{T} = \frac{1}{1.6 \times 10^{-15}} = 0.625 \times 10^{15}$$

$$\Rightarrow f = 6.25 \times 10^{-1} \times 10^{15}$$

$$\Rightarrow \boxed{f = 6.25 \times 10^{14}\ Hz}$$

---

(4)

no. of oscillation $= n = 40$

time $= t = 4$ sec

Wave-length $= \lambda = 10cm = \frac{10}{100} = 0.1m$

Time-Period $= T = ?$

Frequency $= f = ?$

velocity $= v = ?$

Since

$$f = \frac{n}{t} = \frac{40}{4} = 10$$

$$\boxed{f = 10\ Hz}$$

$$T = \frac{1}{f} = \frac{1}{10} = 0.1$$

$$\Rightarrow \boxed{T = 0.1\ sec}$$

$$v = f\lambda = 10 \times 0.1$$

$$\Rightarrow \boxed{V = 1\ m/sec}$$

5 length $= l = 1m$

Time-Period $= T = 2\,sec$

Acceleration due to gravity $= g = ?$

Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

squaring both sides, we get

$$T^2 = 4\pi^2\frac{l}{g} \Rightarrow gT^2 = 4\pi^2 l$$

$$\Rightarrow g = \frac{4\pi^2 l}{T^2} = \frac{\cancel{4}\times(3.14)^2\times 1}{\cancel{(2)}^2}$$

$$\Rightarrow \boxed{g = 9.8596\ m/sec^2}$$

---

6 length $= l = 4mm = 4\times10^{-3}\,m$

Time-Period $= T = ?$

Frequency $= f = ?$

Since

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\times3.14\sqrt{\frac{4\times10^{-3}}{9.8}}$$

$$\Rightarrow T = 6.28\sqrt{0.408\times10^{-3}} = 6.28\sqrt{4.08\times10^{-4}}$$

$$\Rightarrow T = 6.28\,(2.01\times10^{-2})$$

$$= 12.6228\times10^{-2} = 0.126228\times\cancel{10^{1}\times10^{-2}}$$

$$\boxed{T = 0.126228\ sec}$$

$$f = \frac{1}{T} = \frac{1}{0.1262} = 7.9\ Hz$$

$$\boxed{f = 7.9\ Hz}$$

(7)

mass $= m_1 = 125g = \frac{125}{1000} = 0.125\,kg$

Frequency $= f_1 = 2.6$ Hz

Reduced mass $= m_2 = 50g = \frac{50}{1000} = 0.05\,kg$

Frequency $= f_2 = ?$

Since

$$f_1 = \frac{1}{T_1} = \frac{1}{2\pi\sqrt{\frac{m_1}{K}}} = \frac{1}{2\pi}\sqrt{\frac{K}{m_1}}$$

$$\Rightarrow f_1 = \frac{1}{2\pi}\sqrt{\frac{K}{m_1}}$$

$$\Rightarrow f_1^2 = \frac{1}{4\pi^2} \cdot \frac{K}{m_1} \quad \because \text{squaring}$$

$$\Rightarrow K = f_1^2 \cdot 4\pi^2 \cdot m_1 = (2.6)^2 \times 4 \times (3.14)^2 \times 0.125$$

$$\Rightarrow K = 33.32$$

Similarly

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{K}{m_2}} = \frac{1}{2 \times 3.14}\sqrt{\frac{33.32}{0.05}}$$

$$f_2 = \frac{1}{6.28}\sqrt{666.4} = \frac{1}{6.28} \times 25.81$$

$$\boxed{f_2 = 4.11 \text{ Hz}}$$